# An Object-Oriented Framework for SE(3) Motion Planning in Multibody Systems

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**Abstract**

Motion planning in three-dimensional space is a well-understood problem, for which a number of well-established methods exist. Nevertheless, spatial motion planning is a still active topic of research with several open engineering applications to solve. Examples are the optimization of palletizing processes with sensitive goods, the generation of psychophysically realistic motions for ride simulators, and the design of roller-coaster tracks featuring ride effects under fulfillment of physiological and physical constraints.

In this paper, an object-oriented environment for the solution of such problems is presented. As a first step, the problem of smooth interpolation of spatial rotation is discussed. This is done by a quintic spline that allows for continuous angular accelerations of the travelling Frenet frame. The spline description is adapted into a generalized "spline joint" that can be used kinematically and dynamically as an elementary joint, i.e. to form kinematical loops with other bodies as well as generalized equations of motion involving the track coordinate. Different parametrizations of the travelling frame motion along the curve are realized as 'flavours' of the spline joint which can be selected freely and in any combination by the user. For the motion along the trajectory, path planning algorithms or a multibody solution of the corresponding dynamical equations can be used. Optimization of track trajectory is realized by SQP optimization together with morphing techniques while allowing for local user editing. Curve fitting is performed with the Dierckx routine `concur` [1], which computes a suitable knot vector $\underline{\lambda}$ and the corresponding spline coefficients $\underline{c}_j$ along the complete curve by minimizing the sum of "jumps" in the highest derivative so as to obtain the globally "smoothest" curve. In this setting, the shape of the curve is prescribed by the designer (or an optimization algorithm) using control points that are approximated by the curve. The developed environment considers the most important design criteria such as minimal track curvature, avoidance of collisions between track and environment, allowed accelerations, wheel loads on rails and support structure or minimal velocities. Using this approach, a significantly reduced design effort can be achieved as compared with conventional approaches using several simulation tools and CAD-environments. We show the application of these methods for the industrial roller-coaster design environment XTRAC [2].

With the spline joint as an efficient spatial motion interpolation tool, time-optimal motions along prescribed trajectories under dynamic constraints can be addressed. Starting from the work in [3], the basic idea is to map the multibody differential equations to the one-dimensional motion along the prescribed path, and to find at each point of the path the maximally allowed acceleration such that functional limits of the mechanical system will not be violated along the complete trajectory, and that the guided body reaches its end position in minimal time and

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with vanishing velocity. This can be realized by finding the switching points in the $s$-$\dot{s}$ plane ($s$ being the path parameter), where limits such as maximal actuator forces can be represented by continuous (acceleration) limiting curves. Shin and McKay [4] extended this method including the case of velocity proportional actuator forces, and Pfeiffer and Johanni [5] later optimized the algorithm by introducing the concepts of sinks and sources along the upper acceleration limiting curve and considering also the case in which the reduced mass coefficient at the actuator degree of freedom can become singular, leading to critical points. Recent formulations reformulate the problem as a convex optimal control problem [6]. These alternative solution approaches consider only dynamic constraints that can be formulated as a set of linear equations in $\ddot{s}$ and $\dot{s}^2$, and rely on the convexity of the problem formulation.

The talk describes the generalization of these robotic approaches to arbitrary multiloop systems with general rigid-body motion interpolation options, as well as for new constraint types such sticking friction and global power limits [7].

Based on these methods, also time-optimal paths between two configurations can be determined. The flexibility of the method is shown by examples from mining industry and robotics.

**Keywords:** Spatial motion planning, roller-coaster design, time-optimal motion planning, trajectory optimization

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