Investigation of Autonomous Oscillating Linkages[1]

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Abstract

This paper demonstrates the investigation of autonomous motion, in special autonomous oscillation of plane mechanisms, by using two computer-programs in combination with common software. The first part gives a short introduction with respect to the first computer-program, which can be installed at any notebook. This program allows the investigation of plane mechanisms up to 20 links, starting from the type-synthesis till the analysis; considering geometrical, differential-geometrical, kinematical and kinetostatical tasks. The definition of elementary groups (EGs) includes the specification of a link-fixed coordinate system at any link. On the basis of explicit equations the solution of significant tasks is included to this restricted universal computer program. The second part of the paper demonstrates the advantages in researching special topics of autonomous motion of an oscillating slider-crank mechanism by using the announced two computer-programs.

Keywords: Mechanism, linkage, elementary group (EG), prime core-structure, position geometry, differential geometry, kinetics, kinematics, position-parameters, mass-parameters, force-parameters, type-synthesis, autonomous motion, autonomous oscillation, generalized force, kinetic energy, time-calculation, position of equilibrium PE, stationary motion.

1. Introduction

Since the time Computers and special software are available, it is possible to investigate linkages numerically. Graphical simulation by ruler und circle has only the power to investigate coupler curves, geometrical polodes and some other differential geometric variables in special positions. Graphical investigation of motion is impossible. Therefore numerical simulation is a much better and only usable tool. This paper demonstrates the numerical investigation of autonomous oscillation of a plane linkage. As there is up to now no software available, investigation is done by using two different programs.

The first program is a nearly universal program for linkage investigation. Using this program it is very easy to assemble linkages of a tremendous number of types by so called EGs (Elementary Groups). These EGs are defined, described and named. As every linkage type is a unique assemblage of a so called prime core structure and any number of EGs, there name is a combination of the name of the EG and the numbers of links of the core structure at which the links of the EG are fastened. This numbers are separated by hyphens [1], [2].

2. Evolution

Two centuries back and later, the first scientists in the field of mechanisms theory, e.g. Leonhardt Euler, Philipp De La Hire, Franz Reuleaux and Ludwig Burmester investigated the properties of linkages on the basis of so called geometrical motion and till now the education of students is going ahead in this direction. This way renders the investigation and can guide to wrong results. An important mistake exists

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[1] This paper is dedicated to Prof. Dr. K. Lakshminarayana, in memoriam.

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in that facts, to call the differential-quotient 1.order of the spot-vector to a coupler-point, by the independent variable <velocity> and the differential-quotient of 2.order <acceleration>. The so called <kinematics> is exactly only differential geometry, camouflaged by ω = const. Ingenious and expedient is those guideline of research, which considers the properties of mechanisms in connection to the changing values of an independent variable and divide the attributes into: 1. position- and differential-geometrically, 2. mass-geometrically and 3. force-geometrically ones. The courses of corresponding periodical variables demonstrate, how powerful the concerning property is marked by the chosen parameter-values.

Since computers, in special notebooks, and graphical screens are available, also software has been developed for the mathematical investigation of mechanisms. Because it is not possible, to create an absolutely universal program; up to now several programs with different capacity and power have been developed. If these programs are absolutely faultless, the authors could identify this only by one program. Using this tool the properties and motion-behaviour of linkages can be investigated quickly.

In 1924 first time Ferdinand Wittenbauer investigated the motion-behaviour of slider crank mechanisms of a combustion engine graphically. He has demonstrated his results as the so called <Massenwuchtdiagramm>, published in [3]. Richard Grammel tried to investigate the motion behaviour of linkages in star-combustion engines by using electro-mechanical calculators during 1934 till 1945, but without success [4]. First time in 1960 the motion-behaviour of a four bar linkage with <constant kinetic energy> has been calculated by using a computer with special software [5]. In 1975 E. O. Krämer [6] has also investigated by computer the <autonomous motion with constant sum of energy> but in a restricted form; see also the paper of Lakshminarayana and Dizioglu in [7]. Other papers to this topic till now are unknown to the authors. Moreover no software is available, to investigate such motion-behaviour directly without creating additional programming.

3. Autonomous Motion

The topic of this paper demonstrates the mathematical investigation of autonomous oscillating linkages by using two computer-programs. The results are demonstrated in diagrams, including detailed discussions. Because of friction in the joints autonomous motions never can be investigated experimentally. Nevertheless mathematical investigation to this subject is very important; the results make it easy, to understand the specialities of nonlinear mechanism-dynamics. Because up to now the attributes of linkages have been adapted according to any purpose of application, by changing only data of position-parameters, the demanded reliability mostly was not fulfilled. The always critical and with respect to the durability very important joint-reactions depend only conditionally on the position-parameters, but not alone. During the last century the teaching-course <Getriebetechnik or mechanisms theory> mainly has taken into consideration position-geometrical and differential-geometrical aspects for several types of linkages; to less for solving important mechanical problems in industry. On the basis of this fact, during the last decades it was tried to solve the mechanical problems electronically. Several wrong strokes demonstrate, that it is impossible to move an especially important mechanical link with high speed electronically so, as a mechanism can do.

Therefore the topic of this paper exists also in the investigation of energetic behaviour in mechanisms, which shall be demonstrated by the results of oscillating motions. The connection between motion-behaviour of linkages and their mass-geometrical incl. force-geometrical properties shall be investigated. Linkages move autonomous, if only conservative forces and/or moments are impressed to their links. Conservative forces are: the gravity forces and the forces of elastic tension- and/or pressure-springs.

Torsion springs impress conservative moments to the links. The mechanical work, done by conservative forces and moments change only the distribution of energy, but not the sum.

Example: An up to now so called slider crank mechanism, fig. 1, is the object of this investigation, it is an assembly of prime core structure and an EG A2.3, see also fig. 2. Therefore the name of every linkage of this type is A2.3-1-0. The data-dimensions are [mm], [°], [kg] and [kgm^2] for the following parameters:

$\xi_{1.2}$ =50; $\eta_{1.2} = 0$; $l_2 = 100$; $h_{2.3\text{-}0;3} = h_{0.1\text{-}2;3} = 20$; $\delta_{0;3} = 0°$,
So-called position-parameters and

m_1 =0,6; $\xi_{S1} = 50$; $\eta_{S1} = 0$; $\theta_1 = 10850$; m_2 =2,5; $\xi_{S2} = -50$; $\eta_{S2} = 0$; $\theta_2 = 29750$; m_3 =5; $\xi_{S3} = \eta_{S3} = 0$; $\theta_3 = 0{,}001$
So-called mass-distribution-parameters; linkage-motion occurs in horizontal plane.

Data-range and course of certain variables demonstrate the attributes and motion-behaviour of linkages. As we adapt linkages for any purpose changing the parameter-data, we are obliged to control the result. Therefore we investigate the autonomous oscillation of a linkage, because it is very easy to change the motion-conditions by changing the positions of equilibrium PE. The most important property of totally constrained mechanisms consists in that fact that all links only can move together. A special attribute exists in that matter of fact, that the one-sense, equal-incremented changing of the crank-angle will be transformed also into an one-sense but periodically unequal-incremented changing of the turning angle of another link. The motion-behaviour of any linkage becomes obvious in the different courses of velocity and acceleration of the crank angle. This happens, because the mechanical work, done by the impressed forces and moments, changes the kinetic energy.

From the kinetic point of view, linkages of any type and parameter-data are flywheels with variable moment of inertia, which rotates together with the crank (1). The generalized mass (moment of inertia) depends on the position of the independent variable φ_1, crank-angle relative to the frame (0). The differential-quotient of 1.order of the generalized mass by φ_1 is called generalized mass of 1.order. The potential energy of all conservative forces and moments depends also on φ_1.

By the following investigation the values of the used force-geometrical and mass-geometrical variables have been calculated with a computer-program, which uses special standard-tasks. The pictures 2-15 demonstrate the application of this program step by step. Moreover the energy-changing both between the links and between the translational and rotational moving masses of the links must be taken into consideration; the corresponding number-tables have been transferred to the other program.

With the second program the values of KTF1 (velocity) and KTF2 (acceleration) of the independent variable can be calculated and also all other variables depending on it, e.g. for the chosen number of 241 PE (positions of equilibrium). Several results will be demonstrated in diagrams.

The explicit equations are:

$$\dot{\varphi}_1 = \varphi_1^{[1]} = \sqrt{2E/R} \qquad (1)$$

for the KTF1 (Kinematical-Transfer-Function of 1.order).

$$\ddot{\varphi}_1 = \varphi_1^{[2]} = \left(Q - R^{(1)}\, E/R\right)\!/R \qquad (2)$$

for the KTF2 (Kinematical-Transfer-Function of 2.order) of the independent variable of the moving mechanism.

$$\ddot{\varphi}_1 = \varphi_1^{[2]} = Q/R \quad (3)$$

for the KTF2 of the independent variable of the moving mechanism; in the reversal-positions (E=0) and in zero-positions of the generalized mass ($R^{(1)}$) of 1.order.

$$t = \int d\varphi_1 / \varphi_1^{[1]} \quad (4)$$

for the time t, which is needed, oscillating from the starting-position to the respectively arrived position and

$$E_k = R_k \varphi_1^{[1]2}/2; \; E_{k,m} = R_{k,m}\varphi_1^{[1]2}/2; \; E_{k,\theta} = R_{k,\theta}\varphi_1^{[1]2}/2 \quad (5)$$

for the kinetic energy of any link (k) respectively of its translational or rotational moving mass. In these equations represent: m [kg] the mass, θ [kgm^2] the moment of inertia around a perpendicular axis through the centre of gravity of any link, $\varphi_1^{[1]}$ the KTF1, $\varphi_1^{[2]}$ the KTF2 of the independent variable, E the kinetic energy, Q the generalized force, t the time, R the generalized mass and $R^{(1)}$ the generalized mass of 1.order of the mechanism, R_k the generalized mass of link (k).

According to equation (1) the course of KTF1 of the independent variable has extreme values in those positions, in which the quotient E/R is extreme. Considering the PE, this is demonstrated for one position, see fig. 22, or in three positions, see fig. 26, 30 and 34. According to equation (2) the positive KTF2 increases in that negative position-range of oscillation, in which the generalized mass of 1.order is also negative. The extreme value of KTF2 can be greater than the value in reversal position, fig. 26 and 33. The same exists in the positive position-range, where the generalized mass of 1.order is positive and KTF2 becomes negative, fig. 26 and 33. If the generalized mass of 1.order is positive in the negative position-range of oscillation, than the KTF2 can become negative, fig. 26, 30 and 33. The same exists in the positive position-range, if the generalized mass of 1.order becomes negative, than the KTF2 can become positive, fig. 26 and 30.

Equation (3) defines: Although the amount of the generalized force is exactly equal in the reversal-positions, fig. 18, the amount of KTF2 in these positions can only be equal, if the generalized mass is it exactly too. This fact exists, because of the symmetrical course of the generalized mass (fig. 16) in the PE 0° and 180°, see fig. 20, 22, 24, 26; it also exists in two separated positions, see fig. 32; but not in all other positions. According to equation (3) the KTF2 of the resting and of the moving mechanism must be equal: 1.) in the reversal positions of oscillation, because there KFT1 of the independent variable is zero, and 2.) in zero-positions of the generalized mass of 1.order, see fig. 33.

Synopsis of all figures:

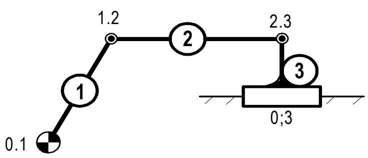

fig. 1: Mechanism of type **A2.3**-1-0, symbolical structure

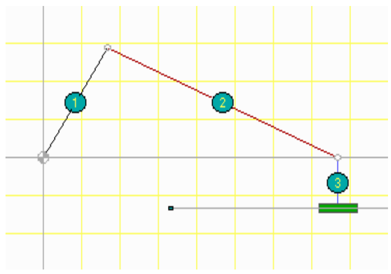

fig. 2: Mechanism of type **A2.3**-1-0 in a scale-representation, animation - position 60°.

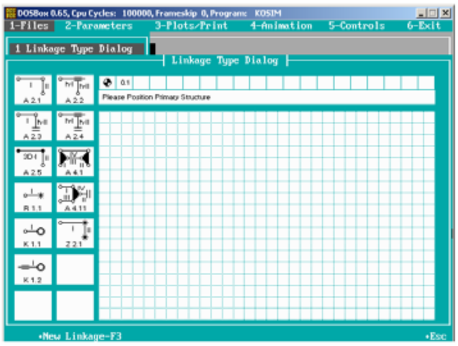

fig. 3: Screen after starting the Type Dialogue, incl. the menu of Elementary Groups (EGs).

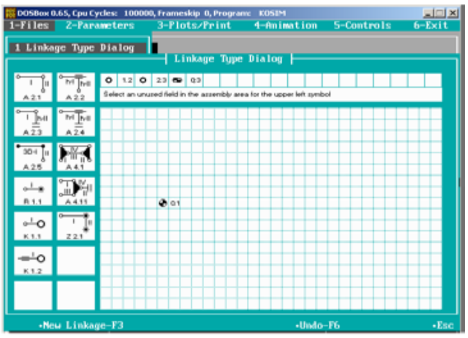

fig.4: Screen after defining the connection of link (0) to link (II) of the EG **A2.3** .

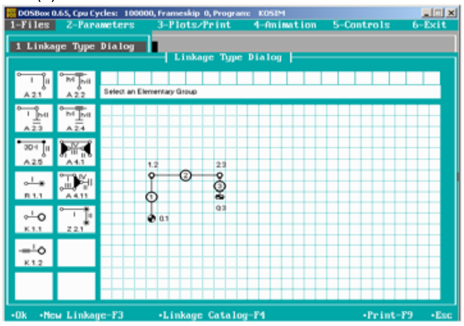

fig. 5: Screen after defining all joints and denotations at the dialogue-mask, abstract structure-sketch.

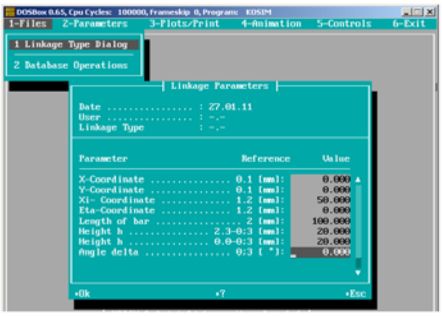

fig.6: Screen after input the position-parameter-data.

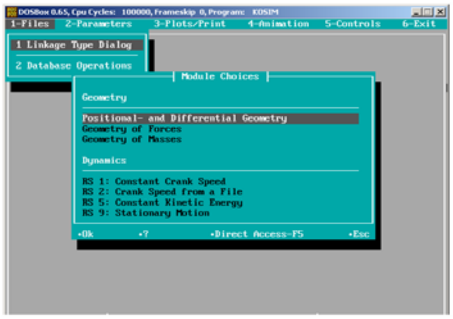

fig.7: Screen with mask for Module Choices: Geometry or Dynamics.

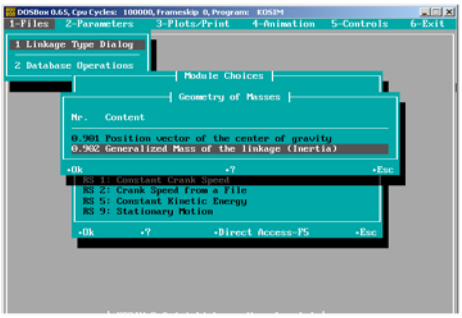

fig.8: Screen with mask Geometry of Masses for choosing the task <Generalized Mass of the Linkage>.

fig.9: Screen after input of <Mass-distribution-parameter-data>.

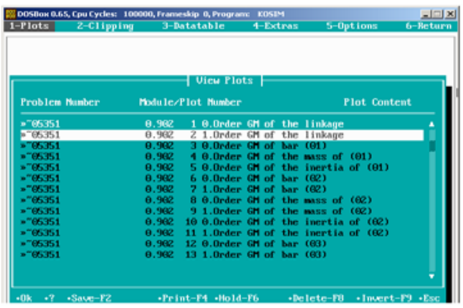

fig.10: Screen with mask <Module/Plot Number> incl. the list of highlighted variables, to be stored.

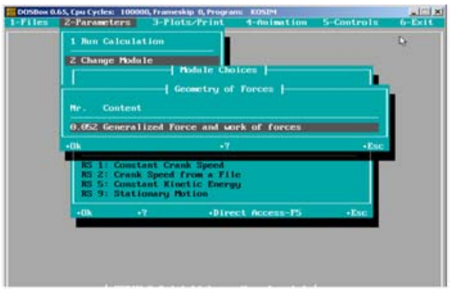

fig.11: Screen with mask <Geometry of Forces> and the marked standard-task <Generalized Force and work of forces>.

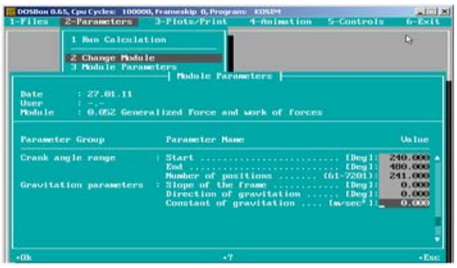

fig.12: Screen with mask <Generalized Force and work of forces> including limited values of the position-range and number of positions to be calculated.

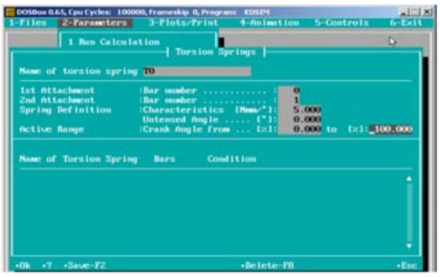

fig.13: Screen with mask <Torsion Spring> and the date-address T0, incl. the characteristics.

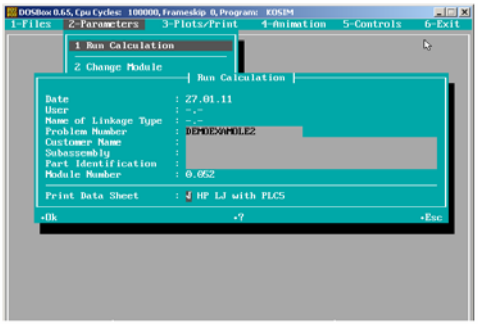

fig.14: Screen with the mask <Run Calculation> for starting the calculation and printing the date-sheets

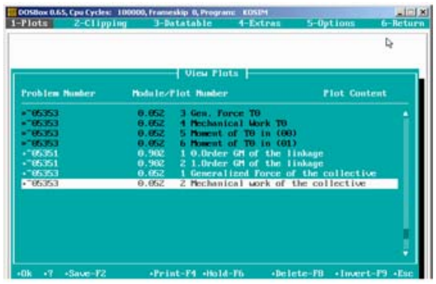

fig. 15: Screen with mask <Module/Plot Number> incl. the list of highlighted variables, which can be stored.

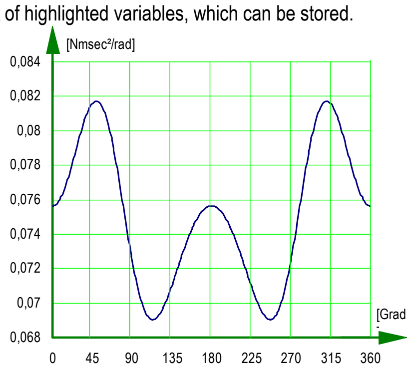

fig. 16: Generalized mass of the mechanism in the whole position-range.

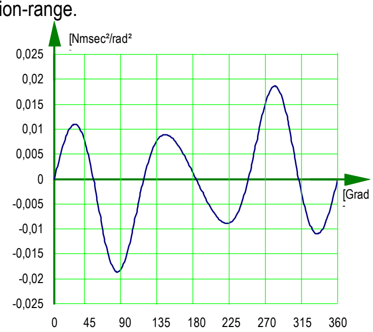

fig. 17: Generalized mass 1.order of the mechanism in the whole position-range.

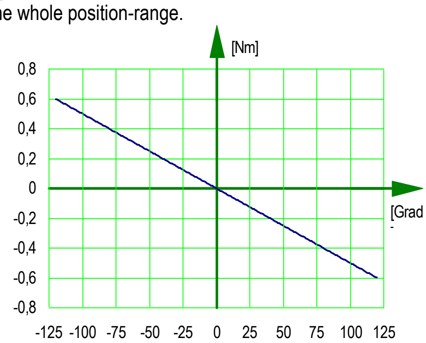

fig.18: Generalized force of the torsion spring in the position -range of oscillation.

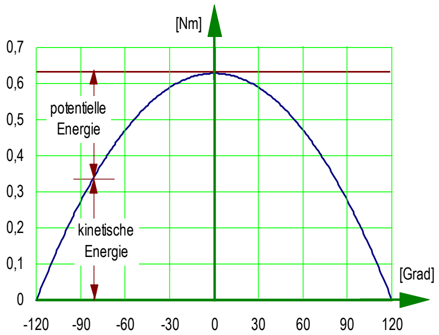

fig. 19: Changing of kinetic and potential energy in the position-range of oscillation.

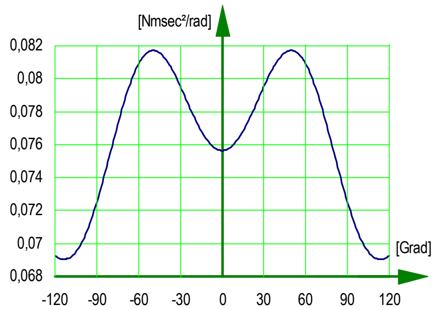

fig. 20: Course of the generalized mass of **A2.3**-1-0 for one oscillation with 120° amplitude by PE 0°.

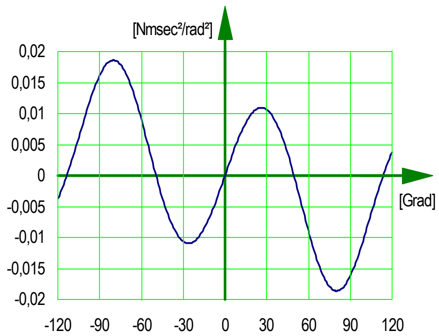

fig. 21: Course of the generalized mass 1.order of **A2.3**-1-0 in the range of one oscillation with 120° amplitude by PE 0°.

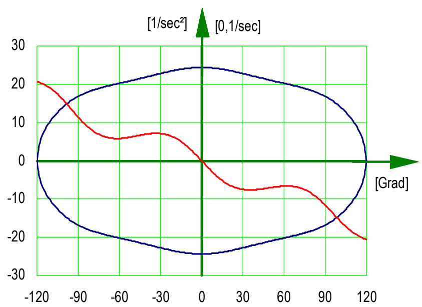

fig. 22: Course of angular velocity and acceleration as function of one oscillation with 120° amplitude by PE 0°.

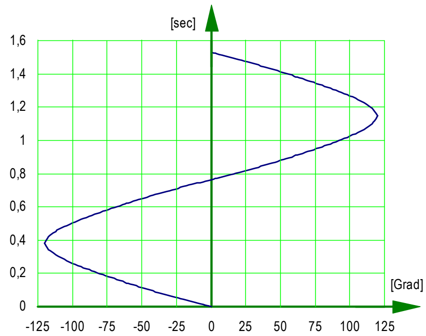

fig. 23: Course of time as function of one oscillation with 120° amplitude by PE 0°.

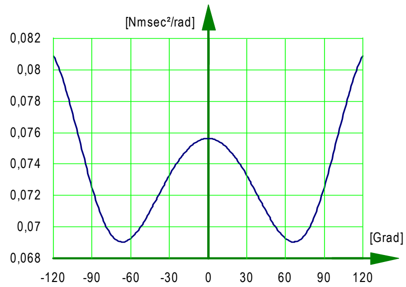

fig.: 24: Course of the generalized mass of **A2.3**-1-0 in the position range of one oscillation with 120° amplitude by PE 180°.

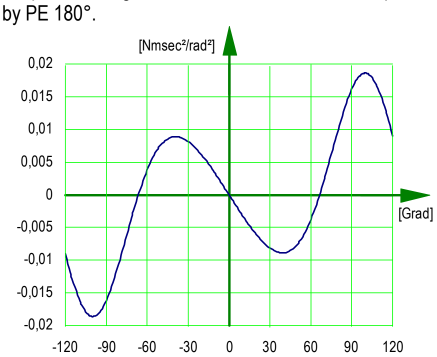

fig. 25: Course of the generalized mass 1.order of **A2.3**-1-0 for one oscillation with 120° amplitude by PE 180°.

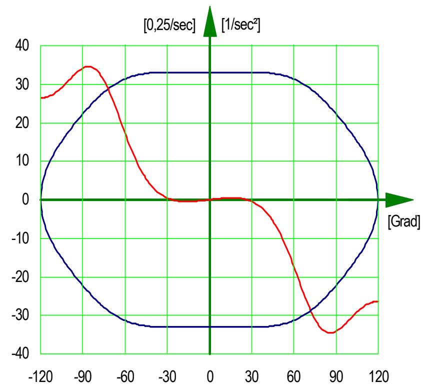

fig. 26: Course of angular velocity and angular acceleration by an oscillation with 120° amplitude and PE 180°.

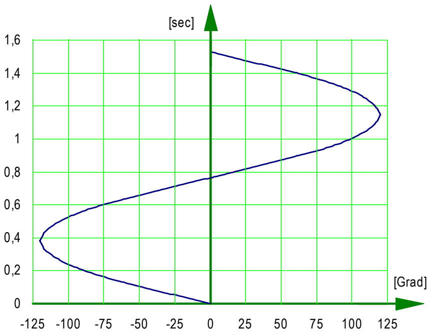

fig. 27: Course of time as function of one oscillation with 120° amplitude by PE 180°.

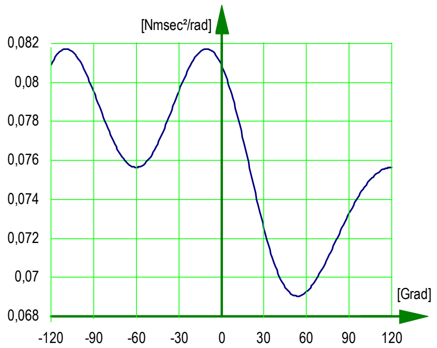

fig. 28: Course of the generalized mass of **A2.3**-1-0 for one oscillation with 120° amplitude by PE 60°.

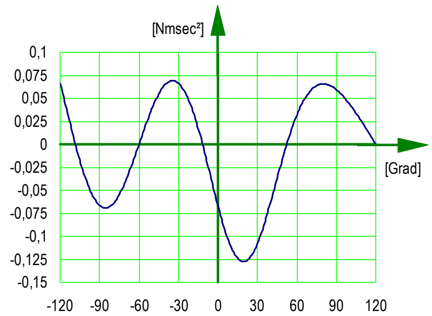

fig. 29: Course of the generalized mass 1.order of **A2.3**-1-0 in the range of one oscillation with 120° amplitude by PE 45°.

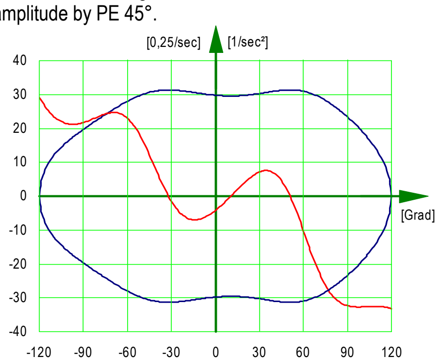

fig. 30: Course of angular velocity and acceleration as function of one oscillation with 120°amplitude by PE 45°.

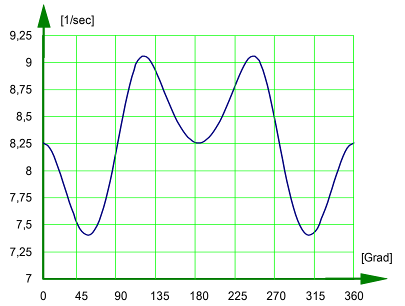

fig. 31: Course of angular velocity of φ_1 in all PE within the range of 0°-360° for oscillating motion with 120° amplitude

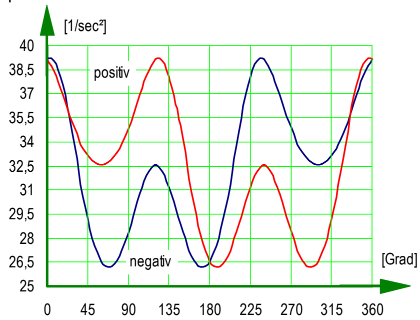

fig. 32: Amount of the acceleration of φ_1 in the reversal positions for all PE within the range of 0°-360° for oscillating motion with 120°amplitude.

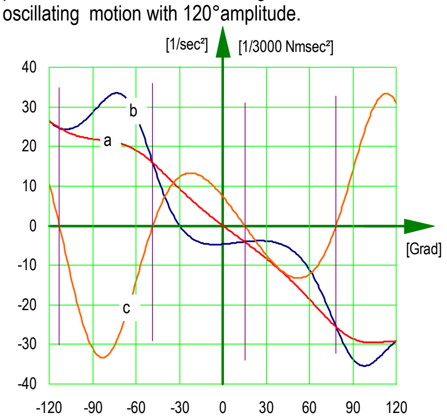

fig. 33: Acceleration of φ_1, caused by the generalized force, with 120° amplitude by PE 0°, **a** for resting, **b** for oscillating linkage, **c** for generalized mass of 1.order.

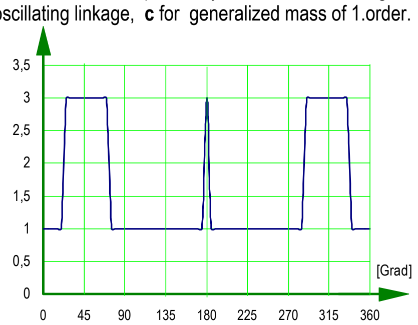

fig. 34: Nadirs of $\varphi_1^{[2]}$ in any position-range with 120° amplitude and PE from 0° - 360°.

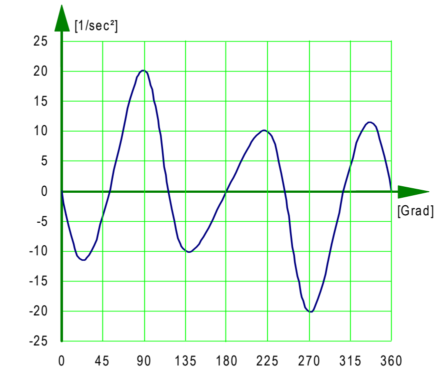

fig. 35: Angular-acceleration of the independent variable by oscillation with 120° amplitude, PE from 0° - 360°.

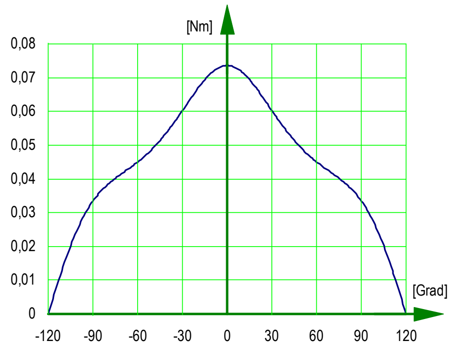

fig. 39: Kinetic energy of link (1) for one oscillation with 120° amplitude by PE 0°.

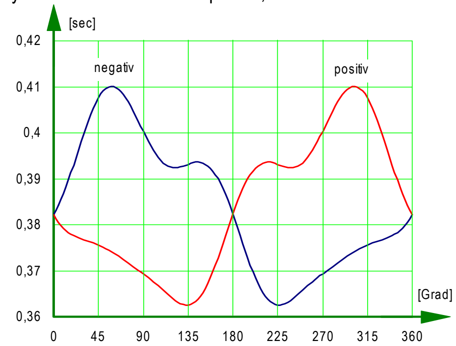

fig. 36: Duration of the oscillations in negative and positive direction for all PE within the range 0°-360° for oscillating motion with 120° amplitude.

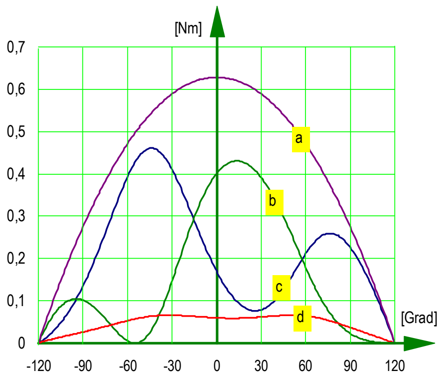

fig. 40: Course **a** of the kinetic energy of linkage **A2.3**-1-0, **b** of link (2), **c** of link (3) and **d** of link (1) for an oscillation with 120° amplitude by PE 45°.

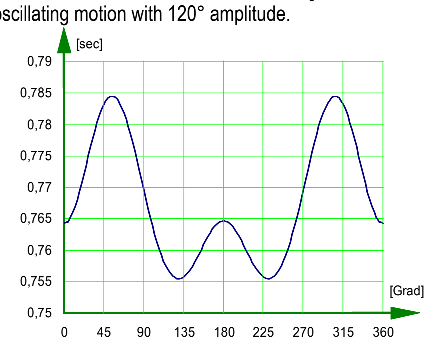

fig. 37: Duration of one total oscillation around all PE within the range 0°-360° for oscillating motion with 120° amplitude.

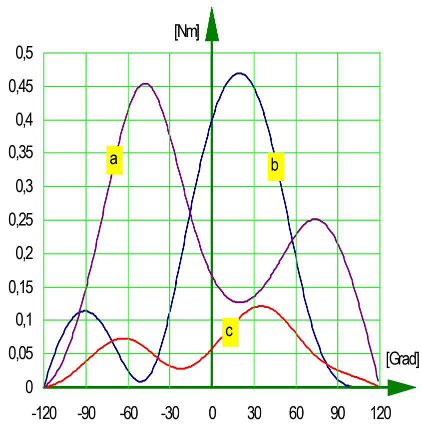

fig. 41: Difference between whole the energy of **A2.3**-1-0 minus kinetic energy of: **a** link (1) and (2), **b** link (1) and (3), and **c** link (2) and (3).

fig. 38: Kinetic energy of link (3) for one oscillation with 120° amplitude by PE 0°.

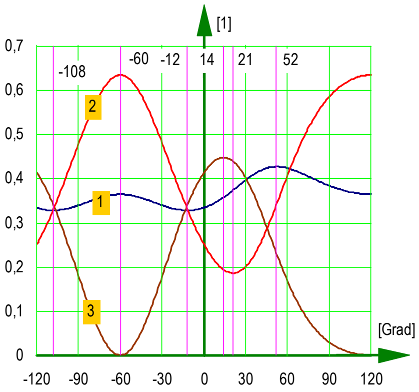

fig. 42: Parcels of the links (1), (2) and (3) at the generalized mass of the linkage in the position range of one oscillation with 120° amplitude by PE 60°.

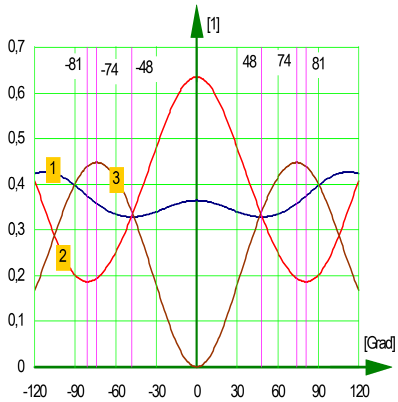

fig. 43: Parcels of the links (1), (2) and (3) at the generalized mass of the linkage in the position range of one oscillation with 120° amplitude by PE 0°.

The fig. 23, 27, 36 and 37 demonstrate the time-difference for the positive and negative half-oscillation and also for the full-oscillation, whereas the oscillation-time of a linear oscillation is independent of the amplitude and PE. Non-linear oscillation depends strong on both, see fig. 36 and 37.

Fig. 19 demonstrates the changing between, kinetic and potential energy during the oscillation with 120° amplitude, in consequence of the mechanical work of the torsion spring.

Fig. 38 demonstrates the course of kinetic energy of link (3) for one oscillation with 120° amplitude by PE 0°. Analogue demonstrates fig. 39 the course of kinetic energy of link (1) for one oscillation with 120° amplitude by PE 0°.
The distribution of kinetic energy to the different links of a linkage **A2.**3-1-0 with 120° amplitude by PE 45° demonstrates fig. 40. Course a represents the whole kinetic energy of this mechanism. The distribution of energy to the different links delivers the courses **b** of link (2), **c** of link (3) and **d** of link (1).

The difference between the whole energy of **A2.3**-1-0 minus kinetic energy of: **a** link (1) and (2), **b** link (1) and (3) and **c** link (2) and (3) results in the courses **a, b** and **c,** see fig. 41.

Fig.42 demonstrates the parcels of the links (1), (2) and (3) at the generalized mass of the linkage in the position-range of one oscillation with 120° amplitude by PE 60°.

Fig.43 represents the parcels of the generalized mass of the links at the generalized mass of the linkage in the position-range of one oscillation with 120° amplitude by PE 0°. By help of number-tables can be recognized the changing of generalized mass in connection with the corresponding position-ranges. The parcels of generalized mass at the links decreases and increases according to the following position-ranges:

- 0° till 48° → the portions of generalized mass of links (1) and (2) decrease whereas the portion of generalized mass of link (3) increases.
- 48° till 74° → portion of link (2) decreases, whereas the portions of links (1) and (3) increases.
- 74° till 81° → portion of link (1) increase, whereas the portions of link (2) and (3) decreases.
- 81° till 120° → portion of link (2) increases, whereas the portion of link (3) decreases and of link (1) in- and decreases.

- 0° till -48° → portion of link (3) increases, whereas the portions of links (1) and (2) decrease.
- -48° till -74° → portions of links (1) and (3) increase, whereas the portion of link (2) decreases.
- -74° till -81° → portion of link (1) increases, whereas the portions of links (2) and (3) decreases.
- -81° till -120° → portion of link (2) increases, whereas the portion of link (1) in- and decreases and of link (3) decreases.

4. Conclusion

Practical important Motion-conditions of linkages are: Stationary motion, forced by all well known power-engines, Starting motion, Motion-outflow and changing from one Stationary motion to another. Autonomous oscillation of linkages gives the Chance to investigate the mechanical motion-processes comprehensively; moreover the variation of PE directly results in another motion-status.

Change of energy, mechanical work of forces and moments etc. have a big influence on the duration of motion and the motion-process itself. It`s now the time for researchers on TMM – Theory of Machines and Mechanisms, to concentrate on the topics in Mechanism-Mechanics. Durability, reliability and especially function security of linkages depend upon the linear and nonlinear motion-behaviour of linkages, which are parts of machines and aggregates.

During the development of linkage-prototypes the industry needs special devices and software to create good solutions for the given tasks in a relative short time. Computer-Aided-Simulation needs improving existing software and/or creating new one. Another important task is the investigation of <Stationary Motion with friction in the joints> to improve the efficiency of all machines and aggregates.

5. References

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