Frictional Dissipation at a Small Crack under Multiaxial Periodic Stresses

Prasun Jana, Anindya Chatterjee

Abstract

Many materials empirically exhibit rate-independent per-cycle dissipation under periodic loading. Towards developing micromechanically motivated models for damping under general triaxial periodic stress, here we consider small frictional cracks in a linearly elastic solid under arbitrary far-field time-periodic tractions. Detailed simulations in 2D and 3D are conducted using ABAQUS, with both zero- and nonzero-mean loading. A simple pseudostatic spring-block model is also studied analytically. Results from the latter, up to one fitted constant, accurately predict all the simulation results from ABAQUS, suggesting that the entire crack face essentially sticks or slips together. These results may lead, in future work, to useful constitutive relations for damping under triaxial periodic loading.

Keywords: Vibration damping, internal dissipation, friction, microcrack, ABAQUS

1 Introduction

Solid bodies undergoing cyclic deformation lose energy. Some energy is lost to the surrounding atmosphere; some goes into supports, joints, and such other dissipative elements; and the rest is dissipated *inside* the material. In this paper we consider the latter means of dissipation, known as material damping or internal friction. In particular, we consider frictional dissipation due to rubbing between the faces of a small internal crack.

Material damping, i.e., the dissipation within the material, can arise from many complex mechanisms. A detailed phenomenological description of material damping can be found in Lazan's famous book [1]. It has been empirically observed that, in many solids, the energy dissipated per unit volume and per cycle of deformation is proportional to the stress amplitude raised to some power $n \geq 2$, and largely independent of frequency in the low frequency range (say, on the order of 100 Hz or less). This can be written as

$$D_m = J\sigma^n \tag{1}$$

where D_m stands for specific material damping, σ represents a suitable stress amplitude, and J is a material constant. We acknowledge that some authors prefer to use strain (nondimensional) in the power law instead of stress; however, there is no real difference if the stress is normalized using a material constant like Young's modulus, and we use stress. The empirical observation of rate independence and approximate

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power law behavior is old and well known. Frequency independence was reported by Lord Kelvin in 1865 [2]. In 1914, Rowett [3] reported static and dynamic experiments in torsion of steel tubes and observed frequency independent power law dissipation with $n = 3$. Kimball and Lovell in 1927 [4] found $n \approx 2$ and frequency independence for eighteen different materials including metals, celluloid, glass, rubber and wood.

Important questions arise immediately from Eq. (1). For example, what is the most suitable stress amplitude for a given cyclic multiaxial stress state? Is J the same for uniaxial and torsional loading? Is n the same? Why or why not? What is the role of residual stresses in the material, if any? What empirical dissipation rule should one use for a combination of tension and torsion? All these questions become important if, say, we wish to make *a priori* predictions of the modal damping ratios of engineering components using finite element analysis.

Clear and useful answers to the above questions are not yet available in the literature. As a possible approach to these questions, in this paper we consider a single, simple but multiaxial, rate-independent micromechanical dissipation model. Specifically, we consider a small flat crack, with Coulomb friction on the faces, inside a linearly elastic body. Internal dissipation due to this crack under oscillatory far-field stresses is studied in this paper using detailed finite element analyses in ABAQUS. A simple summarizing formula for the cyclic dissipation is obtained, up to a multiplicative constant, using a single spring-block model. A near-perfect match with the finite element results indicates that the entire crack face either sticks or slips essentially as a single degree of freedom system. Our results are checked for accuracy through convergence studies using mesh refinement in space and time; and also through a comparison between 2D (plane stress) and 3D (circular crack) simulations.

In future work, we will attempt to use these results to develop micromechanically based rate independent dissipation formulas for solids under multiaxial cyclic loading.

We mention here that the role of frictional dissipation in cracks has been studied in other contexts elsewhere. For example, Abu Al-Rub and Palazotto [5] computationally studied energy dissipation in ceramic coatings and found that frictional dissipation in microcracks contributes significantly to overall material dissipation. However, their study is purely computational and refers to a specific thin coating application, while ours is geared towards an analytical expression which can be used to develop empirical, but micromechanistically based, models for internal dissipation in bulk solids.

2 Micromechanical Model

We consider a small flat crack with Coulomb friction inside an isotropic linearly elastic solid subjected to triaxial periodic loading. An aligned element with applied traction is shown in Fig. (1) (left). We observe that σ_x, σ_y and τ_{xy} do not cause any relative sliding of the crack faces. Thus, these stress components can be dropped from consideration. Subsequently, by a rotation of the coordinate system about the z-axis, τ_{zx} can be made zero. This leaves a normal stress and a single shear component as possibly contributing to frictional slip-induced dissipation. A two-dimensional representation is shown in the figure (right).

Figure (2) depicts periodic normal and shear tractions acting on the element of Fig. (1) (right). The system is understood to have negligible inertia (alternatively,

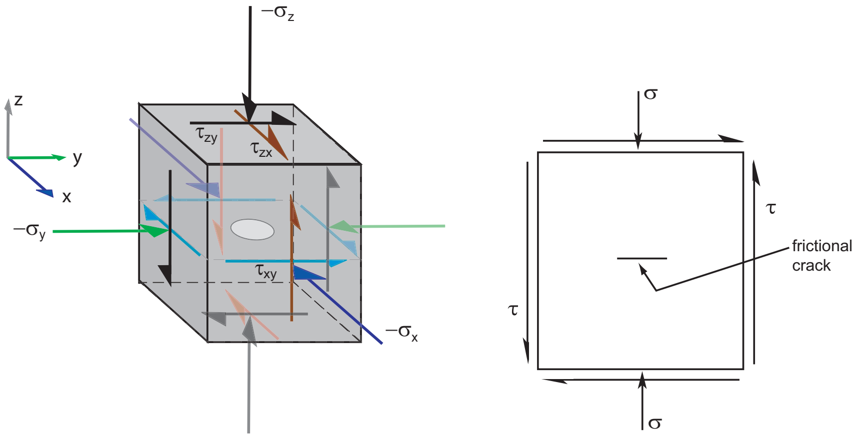


Figure 1: Left: an element with a small embedded frictional crack. Right: 2D representation (note change in sign convention, to take normal compression as positive).

frequencies are low and the response is pseudostatic). Energy dissipation per cycle is automatically frequency-independent for this model. We use a triangular loading pattern, but a sinusoidal one would make no difference.

3 Finite Element Analysis

Finite element analysis of the above system is carried out using ABAQUS Standard, version 6.9. A 4-node bilinear plane stress quadrilateral (CPS4) element is used for a plane stress analysis. The model (see Fig. (1) (right)) is of dimensions $10\,\mathrm{mm} \times 10\,\mathrm{mm}$. The frictional contact (crack) is $1\,\mathrm{mm}$ long and modeled using *hard contact*. Pseudostatic analysis was done, and the energy dissipation computed. Refinement of both mesh size and load steps was used to check convergence. Figure (3) shows an intermediate mesh: final results presented below were from a more refined mesh. Figure (4) shows typical results from the convergence study. Several load cases were then run, and the dissipation results reported in Table 1.

4 Dissipation Formula

We now seek an analytical formula that will capture the results of Table 1. To this end, consider a spring and massless-block system (Fig. (5)) under periodic normal and tangential loading. Define

$$\zeta = \frac{\tau_a}{\sigma_a} \quad \text{and} \quad \beta = \frac{\sigma_m}{\sigma_a}.$$

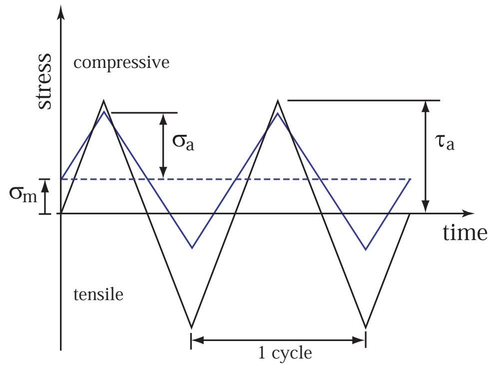


Figure 2: Applied normal and shear loads. Note the sign convention: compressive normal stress is taken positive.

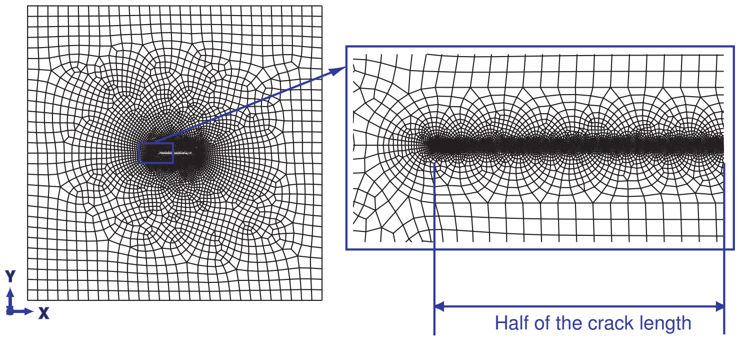


Figure 3: Finite element mesh of 2-D model.

Note that τ_a and σ_a, being amplitudes, are positive by definition; and the mean normal stress σ_m is taken positive when compressive. In all the formulas presented below, the mean shear stress τ_m plays no role, because it affects the mean position of the block but not the steady state cyclic dissipation.

4.1 Case 1: zero mean normal stress ($\beta = 0$)

In each load cycle the crack closes when the normal stress is compressive, and opens when it is tensile. When the crack opens, prior loading history effects disappear. Upon crack closure, sliding (and hence energy dissipation) occurs only if $\tau_a > \mu\sigma_a$ (or $\zeta > \mu$). The steady cyclic energy dissipation for this case can be shown to be

$$D = [\zeta > \mu] \times C\sigma_a^2\mu\zeta \left\{ \frac{\zeta - \mu}{\zeta + \mu} \right\}, \tag{2}$$

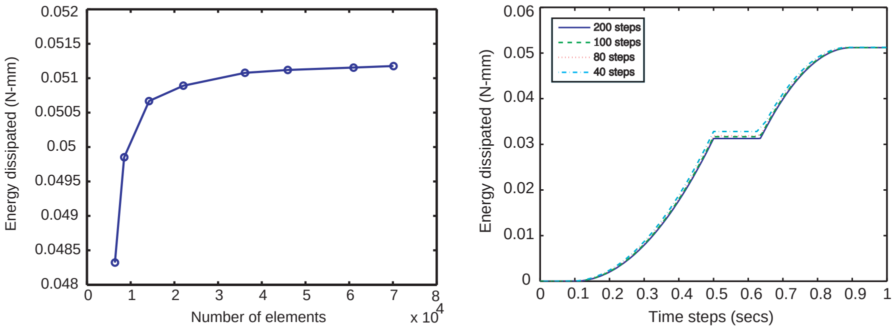


Figure 4: Left: mesh refinement, using 200 time steps in the loading cycle. Right: Effect of changing the number of time steps used.

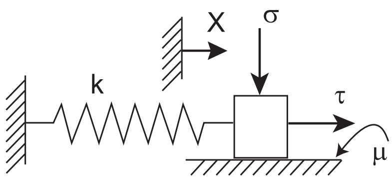


Figure 5: A spring block system under periodic normal and tangential load.

where C is a constant depending on the stiffness and the square brackets denote a logical variable, taking the value 1 if the inequality holds and 0 otherwise.

4.2 Case 2: $-1 < \beta < 1$

For $\beta < -1$ the crack remain always open and there is no dissipation. The case of $-1 < \beta < 1$ is similar to that of case 1, in that the crack opens and closes once in each cycle. The dissipation can be computed using the foregoing result by noting that the instant of crack closure can be used as a reference point, and subsequent stress increments until the next crack opening can be treated in the same way as in Case 1 above. The net result is

$$D = [\zeta > \mu] \times [-1 < \beta < 1] \times C\sigma_a^2 \mu\zeta \left\{ (1+\beta)^2 \frac{\zeta - \mu}{\zeta + \mu} \right\}, \tag{3}$$

where putting $\beta = 0$ recovers Case 1.

4.3 Case 3: $\beta > 1$

If $\beta > 1$, then the crack never opens. To obtain a formula for the steady state cyclic dissipation, the displacement at the instant of minimum compression is first assumed

μ	τ_a (MPa)	σ_a(MPa)	$\beta = \dfrac{\sigma_m}{\sigma_a}$	$\alpha = \dfrac{\tau_m}{\tau_a}$	Dissipation (N-mm)
0.4	100	50	0.6	0	0.05118
0.3	70	30	0.4	0	0.01424
0.3	80	30	0.6	0	0.02201
0.4	110	60	0.4	0	0.04979
0.5	80	40	0.4	0	0.02815
0.4	100	40	1.2	0	0.08281
0.5	80	60	1.1	0	0.07146
0.4	100	100	1.2	0	0.11871
0.3	100	30	1.1	0	0.04957
0.5	90	40	0.6	0	0.04394
0.5	90	40	0.6	0.4	0.04394
0.3	70	30	0.4	0	0.01424
0.3	70	30	0.4	1.2	0.01424
0.4	120	70	-0.4	0	0.01120
0.5	110	80	-0.2	0	0.01962
0.45	95	75	1	0	0.09157
0.35	120	90	1	0	0.13269
0.3	140	100	0	0	0.04060
0.3	140	80	0	0	0.03550
0.5	80	60	2.57	0	0.01129
0.5	80	60	2.45	0	0.02638

Table 1: Dissipation results for various cases in 2D analysis.

to be some x_0; then subsequent displacements are computed over one cycle; and the final displacement is set equal to x_0 again. Solving for x_0, the details of the cycle are known, and the dissipation calculated. The result is

$$D = [\zeta > \mu] \times C\sigma_a^2\mu\zeta \left\{ (1+\beta)^2 \frac{\zeta-\mu}{\zeta+\mu} - (\beta-1)^2 \frac{\zeta+\mu}{\zeta-\mu} \right\}. \tag{4}$$

The above formula holds for $\beta < \dfrac{\zeta}{\mu}$, beyond which there is no cyclic dissipation.

4.4 Single formula

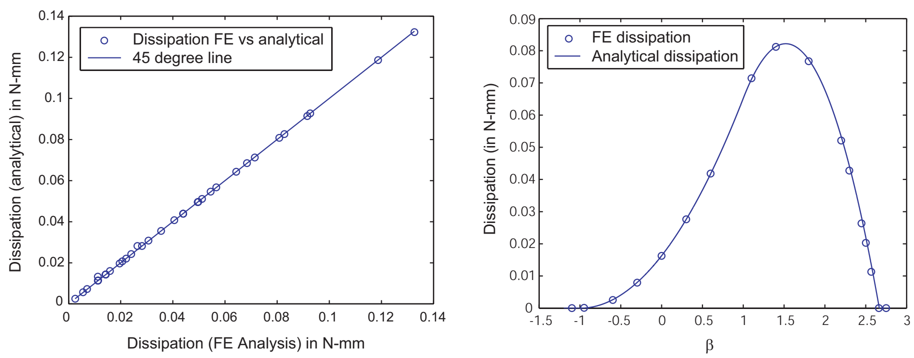
All the foregoing results can be combined into one formula as follows:

$$D = [\beta < \frac{\zeta}{\mu}] \times [\zeta > \mu] \times [\beta > -1] \times C\sigma_a^2\mu\zeta \left\{ (1+\beta)^2 \frac{\zeta-\mu}{\zeta+\mu} - [\beta > 1](\beta-1)^2 \frac{\zeta+\mu}{\zeta-\mu} \right\}, \tag{5}$$

which involves a single load-independent and friction-independent fitted coefficient C. All results for a given crack geometry should hopefully, at least approximately, be described by Eq. (5). We will check the same below.

5 Comparison and Verification

The finite element results of Table 1 are plotted against the dissipation predicted by Eq. (5) with one fitted constant in Fig. (6) (a). The match is excellent, except for a couple of points in the lower dissipation portion of the curve. An understanding of this small apparent mismatch is obtained from a further set of simulations where we fix the parameters $\mu = 0.5$, $\sigma_a = 60$ MPa and $\tau_a = 80$ MPa; the nondimensional ratio $\beta = \sigma_m/\sigma_a$ is varied over a range. Results are shown in Fig. (6) (b). The fitted constant C used in this includes all the new and the old simulation data points (i.e., (a) and (b)). The points with slight mismatch between FE results and the analytical formula lie towards the right side of Fig. (6) (b), where small changes in β lead to large changes in the dissipation. Overall, therefore, it seems clear that the analytical formula is acceptable for practical purposes.



(a) (b)

Figure 6: FE energy dissipation vs. analytical formula (in 2D analysis).

So far, our computational results have been 2D. We also carried out finite element simulation using a 3D solid model, with a flat circular crack. 10-noded tetrahedral elements were used to discretize the volume. Using symmetry, one half of the system was analyzed. The bottom half of that half is shown in Fig. (7). A plot of errors along the line of Fig. (6) (a) shows an excellent match (not given here for reasons of space).

6 Concluding Remarks

We have presented a micromechanical model for internal damping in lightly damped elastic materials. Dissipation occurs due to frictional rubbing at the faces of small flat non-interacting internal cracks within the elastic material.

An analytical expression with one fitted constant has been developed for the energy dissipation under triaxial far field stresses. Finite element analysis results for both 2D plane stress and 3D solids have been used to demonstrate the applicability of the analytical formula.

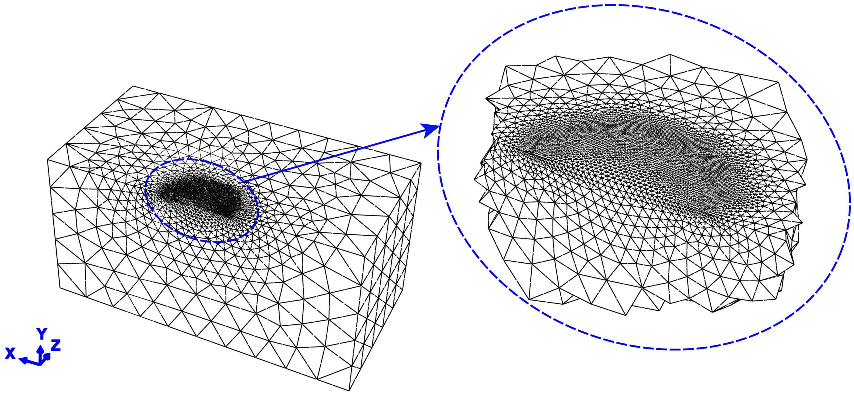


Figure 7: One half of the 3D FE model.

The present micromechanical model is expected to form a building block in a model with randomly dispersed and oriented dissipation sites, whose average dissipative behavior will be taken up in future work.

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